Some Experimental Signatures of the Standing Wave Braneworld

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PLAN OF THE TALK

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- Localization Problem
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Brane models

The scenario where our world is associated with a brane embedded in a higher dimensional space-time **[1]** has attracted a lot of interest with the aim of solving several open questions in modern physics.

Most of brane models were realized as time independent field configurations. Here we present the braneworld scenario with non-stationary metric coefficients proposed in **[2]**. The braneworld is generated by **5D** standing gravitational waves coupled to a scalar field in the bulk.

The nodes of standing waves correspond to **Anti-de-Sitter** 'islands' having the different **4**-dimensional cosmological and gravitational constants,

$$\Lambda_n = e^{2a\xi(n)}\Lambda_n, \quad G_n = e^{-2a\xi(n)} G_5/2d,$$

and could be used to solve the hierarchy and cosmological constant problems.

[1] **N. Arkani-Hamed, S. Dimopoulos** and **G. Dvali**, *Phys Lett.* **B 429** (1998) 263

M. Gogberashvili, *Int. J. Mod. Phys.* **D 11** (2002) 1635, [hep-ph/9812296]

L. Randall and **R. Sundrum,** *Phys. Rev. Lett.* **83** (1999) 3370; 4690.

[2] **M. Gogberashvili** and **D. Singleton,** *Mod. Phys. Lett.* **A 25** (2010) 2131.

Localization problem

A key requirement for theories with extra dimensions is that the various bulk fields be localized on the brane. It is especially economical to consider models with a pure gravitational localization mechanism, since gravity has the unique feature of having a universal coupling with all matter fields. To have localized fields on a brane 'coupling' constants appearing after integration of their **Lagrangians** over the extra coordinates must be non-vanishing and finite.

In **(1+4)**-dimensional models spin-**0** and spin-**2** field are localized on the brane with decreasing and spin-**1/2** field with increasing warp factors **[1]**; spin-**1** field is not normalizable at all **[2]**.

For **(1+5)**-dimensions spin **0**, **1** and **2** fields are localized on the brane with decreasing warp factor and the spin-**1/2** field with increasing warp factor **[3]**.

So to fulfill the localization of **Standard Model** particles in **(1+4)**- or **(1+5)**-spaces it was required to introduce other interaction but gravity. The possibility of pure gravitational trapping of zero modes of all bulk fields, but using artificial matter sources, was demonstrated for the brane solutions with an increasing warp factor in **[4]**.

[1] B. Bajc and **G. Gabadadze,** *Phys. Lett.* **B 474** (2000) 282.
[2] A. Pomarol, *Phys. Lett.* **B 486** (2000) 153.
[3] I. Oda, *Phys. Rev.* **D 62** (2000) 126009.
[4] M. Gogberashvili and **D. Singleton**, *Phys. Rev.* **D 69** (2004) 026004;
M. Gogberashvili and **P. Midodashvili**, *Phys. Lett.* **B 515** (2001) 447.

Mechanical Waves

Sand on a rectangular plate shows the resonance patterns at specific frequencies of oscillation: $\omega \gg m c^2/\hbar$.

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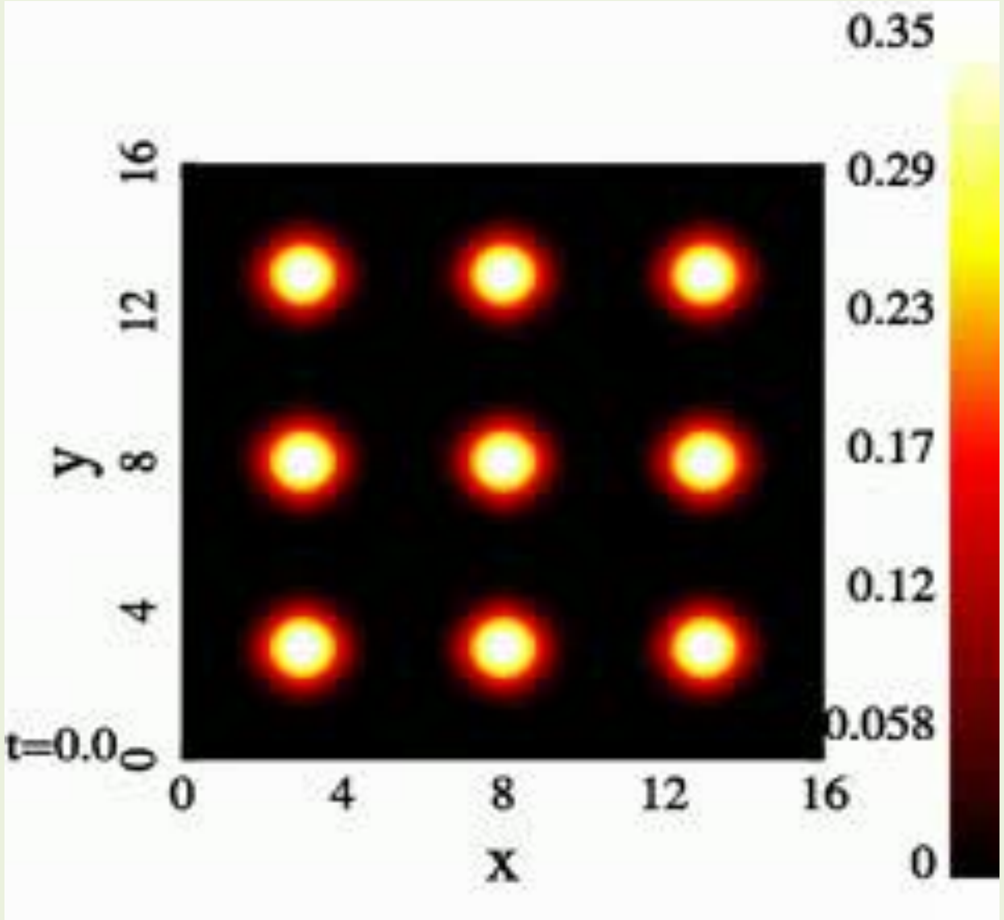
Optical Lattices



Standing electromagnetic waves, ***optical lattices***, can provide trapping of particles (**Nobel** Prize in **1997**) by scattering, dipole **[1]** and quadrupole forces **[2]**.

[1] **H.J. Metcalf** and **P. van der Straten**, *Laser Cooling and Trapping* (Springer, New York 1999).

[2] **N. Moiseyev, M. Šindelka** and **L.S. Cederbaum**, *Phys. Rev.* **A 74** (2006) 053420.

Ghost scalar fields

In the case of **(1+4)**-space with bulk phantom-like massless scalar field, $\sigma(t,r)$, the **5D Einstein** equations, $R_{ab} = - \partial_a\sigma\, \partial_b\sigma + 3/2\, g_{ab}\Lambda_5$, have the standing wave solution [1],

$$ds^2 = e^{2a|r|}(dt^2 - e^u dx^2 - e^u dy^2 - e^{-2u} dz^2) - dr^2 ,$$

$$u(t,r) = A\ \sin(\omega t)\, e^{-2ar}\, Y_2(\omega e^{-ar}/a) ,$$

where $a = (3\Lambda_5/8)^{1/2}$ is the curvature scale, A is a constant and $Y_2(r)$ is the 2^{nd} order **Bessel** function of the second kind.

Standing waves can provide a new universal localization mechanism. Indeed, as it is clear from the equations of motion of spinless particles in the quadrupole approximation [2],

$$DP^\nu/ds = F^\nu = - J^{\alpha\beta\gamma\delta}\, D^\nu R_{\alpha\beta\gamma\delta}/6 ,$$

where P^ν is the total momentum and $J^{\alpha\beta\gamma\delta}$ is the quadrupole moment, the quadrupole force F^ν will eject particles out of the regions with high curvature.

[1] **M. Gogberashvili** and **D. Singleton,** *Mod. Phys. Lett.* **A 25** (2010) 2131;
M. Gogberashvili, *JHEP* **09** (2012) 056.
[2] **W.G. Dixon,** *Nuovo Cim.* **34** (1964) 318; *Gen. Rel. Grav.* **4** (1973) 199.

Boundary conditions

The ghost-like field **$\sigma(t,r)$**, along with the metric oscillations **$u(t,r)$**, must be unobservable on the brane. This condition quantizes the oscillation frequency of the standing wave in terms of the curvature scale:

$$\omega / a = X_n \ ,$$

where **X_n** is the **n**-th zero of the **Bessel** function **$Y_2(r)$**. Correspondingly, the functions **$\sigma(t,r)$** and **$u(t,r)$** vanish at the number of points along the extra dimension. These points - the nodes of standing wave - can be considered as **4D** space-time 'islands', where the matter particles are assumed to be bound. Brane is placed at **$r = 0$**, in one of the nodes of standing waves.

In equations of matter the oscillatory function **$u(t,r)$** enters via exponentials:

$$e^{bu} = \Sigma \, (bu)^n / n! \ .$$

To localize matter fields we suppose that the frequency of standing waves is much larger than frequencies associated with the particles on the brane. In this case we can perform time averaging of oscillating exponents in the equation for matter fields. For the time averages of oscillating exponents we find **[1,2]**:

$$\langle e^{bu} \rangle = I_0\left(|bA| \, e^{-2a|r|} Y_2(e^{a|r|}\omega/a)\right) ,$$

where **$I_0(r)$** is the modified Bessel function of the zero order.

[1] M. Gogberashvili, P.Midodashvili, L.Midodashvili, *Phys. Lett.* **B 702** (2011) 276
[2] M. Gogberashvili, A. Herrera, D. Malagon, *Clas. Quant. Grav.* **29** (2012) 025007.

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Localization of scalars

The action of **5D** massless scalar field has the form:

$$S = - \int d^4x dr\, (g)^{1/2}\, g^{AB} \partial_A \Phi \partial_B \Phi ,$$

with the determinant $(g)^{-1/2} = e^{4a|r|}$. Separating variables,

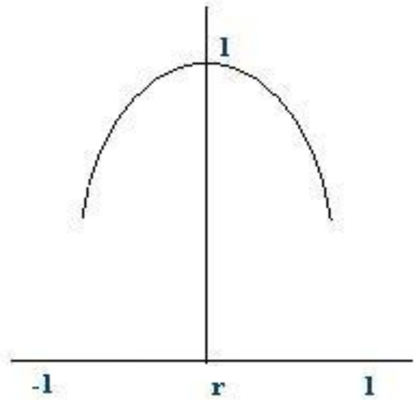
$$\Phi(x^A) = \exp(ip_\alpha x^\alpha)\, \rho(r) ,$$

and time averaging the **Klein-Gordon** equation we find behavior of extra dimension part of the zero mode $\rho(r)$ far from and close to the brane [1,2]:

$$\rho(r)|_{r\to\pm\infty} \sim e^{-4a|r|} ,$$

$$\rho(r)|_{r\to\pm 0} \sim \mathrm{const} .$$

So $\rho(r)$ has maximum at $r = 0$, falls off from the brane, and vanishes at the infinity as $e^{-4a|r|}$. In **S** the determinant and the metric tensor with upper indices give the increasing factor $e^{2a|r|}$. This is the reason why in the original brane models the scalar field zero modes (with the constant extra part) can be localized on the brane only for $a < 0$. In our model $\rho(r)$ is not a constant, moreover, for $a > 0$ it contains the exponentially decreasing factor leading to the convergence of the action integral **S** by **r**.



Shape of $\rho(r)$ close to the brane

[1] **M. Gogberashvili,** *JHEP* **2012** (2012) 56;

[2] **M.Gogberashvili, P.Midodashvili, L.Midodashvili,** *Phys. Lett.* **B 702** (2011) 276.

Localization of vectors

The action of the **5D Abelian** vector field has the form:

$$S = -\int d^4x dr\, (g)^{1/2}\, g^{AB} g^{CD} F_{AC} F_{BD}\,,$$

were $F_{AB} = \partial_A A_B - \partial_B A_A$. We seek for solution in the form:

$$A_t(x^C) = \rho(r)\, a_t(x^\alpha)\,, \quad A_x(x^C) = e^{u(t,r)} \rho(r)\, a_x(x^\alpha)\,, \quad A_y(x^C) = e^{u(t,r)} \rho(r)\, a_y(x^\alpha)\,,$$

$$A_z(x^C) = e^{-2u(t,r)} \rho(r)\, a_z(x^\alpha)\,, \quad A_r(x^C) = 0\,,$$

where $u(t, r)$ is the oscillatory metric function and

$$a_\beta\,(x^\alpha) \sim \varepsilon_\beta \exp(ip_\alpha x^\alpha)$$

denote the components of the **4D** vector potential on the brane. Time averaging the **Maxwell** equations we find behavior of the extra dimension part of vector zero mode $\rho(r)$ far from and close to the brane [1,2]:

$$\rho(r)|_{r\to\pm\infty} \sim e^{-2a|r|}\,, \qquad \rho(r)|_{r\to\pm 0} \sim const\,.$$

So $\rho(r)$ has maximum at $r = 0$, falls off from the brane and vanishes at the infinity as $e^{-2a|r|}$. In S the extra dimension parts of the determinant and two metric tensors with upper indices cancel each other. Because of this in the original brane models the vector field zero modes (with the constant extra part) cannot be localized on the brane for any sign of a. In our model $\rho(r)$ is not a constant, for $a > 0$ it contains the exponentially decreasing factor and integral of S by r is convergent.

[1] **M. Gogberashvili,** *JHEP* **2012** (2012) 56;

[2] **M.Gogberashvili, P.Midodashvili, L.Midodashvili,** *Phys. Lett.* **B 707** (2012) 169.

Localization of spinors

The action of the massless **5D** spinor field has the form:

$$S = - \int d^4x dr\, (g)^{1/2}\, i\Psi\Gamma^A D_A \Psi \,.$$

We perform the chiral decomposition for the bulk fermion field wave function:

$$\Psi(x^C) = \psi_L(x^\alpha)\,\lambda(r) + \psi_R(x^\alpha)\,\rho(r)\,.$$

Here $\lambda(r)$ and $\rho(r)$ are extra dimension factors of left and right fermions and

$$\psi_L(x^\alpha) \sim \varepsilon_L \exp(ip_\alpha x^\alpha)\,,$$
$$\psi_R(x^\alpha) \sim \varepsilon_R \exp(ip_\alpha x^\alpha)\,,$$

We find behavior of extra dimension parts of the left and right fermionic zero modes far from and close to the brane [1,2]:

$$\lambda(r)|_{r=\pm 0} = C\,, \qquad \lambda(r)|_{r\to\pm\infty} \sim e^{-3a|r|}\,,$$
$$\rho(r)|_{r=\pm 0} = 0\,, \qquad \rho(r)|_{r\to\pm\infty} \sim e^{-2a|r|}\,.$$

The extra dimension part of the left spinor $\lambda(r)$ has maximum on the brane and decreases $\sim e^{-3a|r|}$ at the infinity, so left zero modes are localized on the brane. But right fermions are absent on the brane - $\rho(0) = 0$. Also zero modes of right fermions are not normalizable.

[1] **M. Gogberashvili, P. Midodashvili, L. Midodashvili**, *Int. J. Mod. Phys.* **D 21** (2012) 1250081.

[2] **M. Gogberashvili, O. Sakhelashvili, G. Tukhashvili**, *Mod. Phys. Lett.* **A 28** (2013) 1350092.

Asymmetric inflation

If we relax the condition of proportionality of scalar and gravitational fields:

$$\sigma(t,r) \sim u(t,r) - 2Ht\ ,$$

H is a constant, the system of **Einstein** and ghost-like field equations have exact solution corresponding to the standing waves with **t**-depended amplitude:

$$ds^2 = e^{2a|r|}(dt^2 - e^{u-Ht}dx^2 - e^{u-Ht}dy^2 - e^{-2u+2Ht}dz^2) - dr^2\ .$$

The amplitude of scalar waves increases/decreases with **t** depending on **H**.

1) If **H > 0**, the amplitude of **σ(t,r)** decreases, while space expands exponentially in **z** and squeezes in **x-y** directions, brane shrinks into **1**-string.

2) H < 0, the amplitude increases, **z**-distances decrease and **x-y** plane expands.

The mechanism can be relevant for dimensional reduction in string theories **[1]**

As the anisotropic energy of the **3**-brane leaks into the bulk the braneworld isotropizes by itself and the phantom scalar field exponentially vanishes **[2]**.

[1] M. Gogberashvili, A. Herrera-Aguilar, D. Malagón-Morejón and **R.R. Mora-Luna**, *Phys. Lett.* **B 725** (2013) 208.

[2] M. Gogberashvili, A. Herrera-Aguilar, D. Malagón-Morejón, R.R. Mora-Luna and **U. Nucamendi**, *Phys. Rev.* **D 87** (2013) 084059.

Standing GW-s in 4D

For the massless real scalar field, which obeys the **Klein-Gordon** equation,

$$\frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}g^{\alpha\beta}\partial_\beta\varphi\right)=0$$

the **Einstein** equations can be written as:

$$R_{\alpha\beta}=\partial_\alpha\varphi\partial_\beta\varphi$$

We take the metric *ansatz* **[1]**,

$$ds^2=\frac{e^S}{\sqrt{1+k|z|}}\left(dt^2-dz^2\right)-(1+k|z|)\left(e^u dx^2+e^{-u}dy^2\right)$$

which is the combination of the domain wall solution (**$S(t,z) = u(t,z) = 0$**) **[2]** and the plane wave solutions (**$k = 0$**) **[3]**. In our case the solution has the form:

$$\varphi(t,z) = A/2 \sin(\omega t)\, J_0(\omega z + \omega/k)\,, \qquad u(t,z) = A \cos(\omega t)\, J_0(\omega z + \omega/k)\,,$$

$$S(t,z) = A\, \omega^2/4k^2\, (1 + kz)^2\, (J_0^2 + 2J_1^2 - J_0J_2)\,,$$

J_n are **Bessel** functions, **A** is the wave amplitude and **$\omega = k\xi_n$** is its discrete frequency (**ξ_n** are the zeros of **J_0**). Oscillations of **$u(t,z)$** and **$\varphi(t,z)$** are **$\pi/2$** out of phase, i.e. the energy passing between the scalar and gravitational waves.

[1] M. Gogberashvili, S. Myrzakul and **D. Singleton,** *Phys. Rev.* **D 80** (2009) 024040.
[2] A. Vilenkin, *Phys. Rev.* **D 23** (1981) 852.
[3] J. Griffiths, *Colliding Plane Waves in General Relativity* (Oxford Un. Press, 1991).

Small extra space

If the amplitude of the bulk standing waves is small, $A \ll 1$, we can put:

$$u = \varphi = S = 0,$$

and consider the metric *ansatz* without oscillatory metric functions [1]:

$$ds^2 = \frac{1}{(1 - a|r|)^{2/3}}\left(dt^2 - dr^2\right) - (1 - a|r|)^{2/3}\left(dx^2 + dy^2 + dz^2\right).$$

This metric has the horizon at $|r| = 1/a$, where components of the **Ricci** tensor get infinite values, while all gravitational invariants are finite. It resembles the situation with the **Schwarzschild** Black Hole. However, in contrast to the Black Hole case the determinant of this metric

$$\sqrt{g} = (1 - a|r|)^{2/3}$$

becomes zero at $|r| = 1/a$. As the result matter fields are confined in the extra space. To provide experimentally acceptable localization the actual size of the extra dimension must be sufficiently small, $\leq 1/M_H$, where M_H denotes the **Higgs** scale. So the curvature scale a must be large $M_H \leq a \leq M$, where M is the **5D** fundamental scale.

[1] **M. Gogberashvili** and **P. Midodashvili**, *arXive:*1310.1911 [hep-th].

Real bulk scalar field

We consider **5D** space without cosmological constant containing a brane at $|r| = 0$ and a non-self interacting real scalar field $\varphi(t,|r|)$, and use the metric **[1]**:

$$ds^2 = \frac{e^S}{(1 - a|r|)^{2/3}}\left(dt^2 - dr^2\right) - (1 - a|r|)^{2/3}\left(e^u dx^2 + e^u dy^2 + e^{-2u} dz^2\right),$$

where a is a positive constant and $S=S(|r|)$, $u=u(t,|r|)$ are some functions. On the brane $|r| \to 0$ we put the boundary conditions:

$$u|_{r=0} = 0, \quad \varphi|_{r=0} = 0, \quad S|_{r=0} = 0.$$

Then the solutions to the system of **Einstein** and scalar field equations is:

$$u(t,|r|) = A\sin(\omega t) J_0(X),$$

$$\varphi(t,|r|) = \sqrt{3M^3/2}\; A\cos(\omega t) J_0(X),$$

$$S(|r|) = \frac{3}{2}A^2\left\{X^2\left[J_0^2(X) + J_1^2(X) - \frac{1}{X}J_0(X)J_1(X)\right] - X^2(0)J_1^2(Z_n)\right\},$$

where ω denotes the frequensy of standing waves, $X = \omega(1 - a|r|)/a$ and $Z_n = \omega / a$ is the n-th zero of the function $J_0(X)$.

[1] M. Gogberashvili and P. Midodashvili, *arXive:*1310.1911 [hep-th].

Large extra space

For large oscillations, **$A >> 1$**, the width of the brane is determined by **S**, and trapping is provided by bulk oscillations. Consider a real massless scalar field: $\Phi(x^A) = \varphi(x^\nu)\rho(|r|)$, where the **4D** factor obeys the dispersion relation:

$$\eta^{\nu\beta}\partial_\nu\partial_\beta\varphi\left(x^\mu\right) = -m^2\varphi\left(x^\mu\right).$$

Close to the brane the zero mode (**$m = 0$**) wavefunction has the expansion:

$$\rho_0(r)\,|_{|r|\to 0} = C\left(1 - \frac{1}{6}BE^2a|r|^3\right) + O\left(a^4|r|^4\right),$$

At the horizon, **$|r| \to 1/a$**, the expansion of the zero mode is done by:

$$\rho_0(r)\,|_{|r|\to 1/a} = C\left[1 + \frac{9c^2}{4a^2}(1-a|r|)^{2/3}\right] + O\left((1-a|r|)^{4/3}\right),$$

Then the **5D** scalar field action:

$$S_0 = \int dx^5\left[Q_1(r)\partial_t\varphi^2 - Q_2(r)\left(\partial_x\varphi^2 + \partial_y\varphi^2 + \partial_z\varphi^2\right) - Q_3(r)\varphi^2\right],$$

where: $Q_1(r) = (1-a|r|)\rho_0^2,\ Q_2(r) = (1-a|r|)^{-1/3}e^S\rho_0^2,\ Q_3(r) = (1-a|r|)\rho_0'^{\,2},$ is integrable over **r**, and the scalar field zero mode is localized on the brane. When the curvature scale **a**, which determines the size of extra dimension, is smaller than the scale associated with the width of the brane, the **KK** modes of the mass **$m \approx 3.8\,a$**, can be created in accelerators at relatively low energies.

Conclusions

- We find pure gravitational localization mechanism within the 5D standing wave braneworld. The model represents single brane which bounds collective oscillations of gravitational and scalar field standing waves.

- For the bulk ghost-like scalar fields and increasing warp factor we show localization of zero modes of spin-0, -1 and -2 fields. While only the left spin-½ field zero modes are localized on the brane, right fermions are localized in the bulk. The cosmological solution provides with natural mechanisms of the dimensional reduction and brane isotropization.

- For the bulk real scalar fields and when the amplitude of the standing waves is small, matter fields are localized due to the presence of the horizon in extra dimension. When oscillations are large trapping is provided by the pressure of bulk waves. The mass of the lightest KK mode is determined by the smaller scale corresponding to the horizon.