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Gauge invariance - Dynamics of physical
processes

Non-perturbative phenomena in QFT:
 bound states, confinement, spontaneous symmetry
 breaking, mass generation, phase transitions, etc.

Lattice calculation

$$\begin{aligned}
 Z[j(x)] &= \int D\psi(x) e^{iS[\psi(x), j(x)]} \\
 &= \int \prod_i d\psi_i e^{iS(\psi_i, j_i)} \quad \text{Lattice} \\
 &= \sum_n g^n \int D\psi(x) e^{iS_0[\psi(x), j(x)]} \left(\int \psi^N(x) dx \right)^n
 \end{aligned}$$

Continuum QCD **უწყვეტი QCD**

Dynamical equations

Infinite series \rightarrow equation

Continuum QFT:

Argon National Lab.

University of Graz,

Giessen Uni

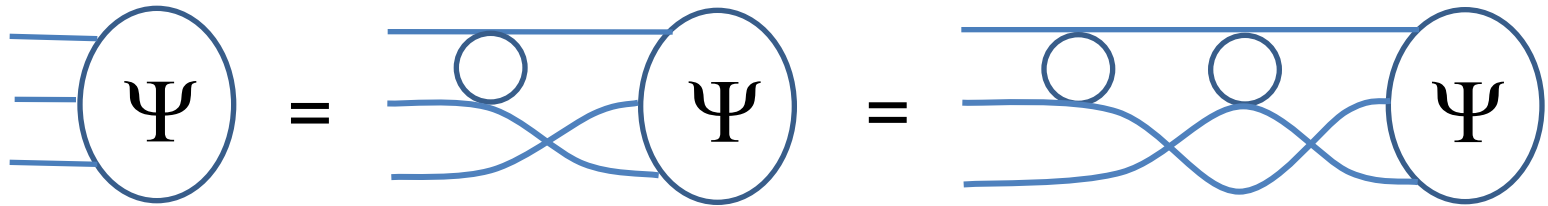
Darmstadt, Institute of Nuclear Physics

Three relativistic particles

$$\psi = G_0(V_{12} + V_{23} + V_{31})\psi \Rightarrow \psi = -G_0 T_1 P_{12} \psi$$

where

$$T_{12} = V_{12} + V_{12} G T_{12}$$



In QFT

$$V_{12} = \text{[Diagram: a square with four arrows pointing outwards from its corners and a small circle below it]} = ?$$

compact kernel

$$\text{[Diagram: a circle with four arrows pointing outwards from its corners]} = T^{(2)}(E - E_3) \rightarrow \int T^{(2)}(E - z) g(z) dz$$

$$\text{[Diagram: a horizontal line]} = g(z) = \frac{1}{z - \sqrt{\vec{p}^2 + m^2} + i\varepsilon}$$

$$\Rightarrow \text{[Diagram: a horizontal line with a small circle in the middle]} = \text{[Diagram: a horizontal line]} + \text{[Diagram: a horizontal line with a dashed loop on top]} + \text{[Diagram: a horizontal line with two dashed loops on top]} + \dots$$

Relativistic four-body problem:

$$V = \sum \left[\begin{array}{c} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \\ \text{---} \\ \text{---} \end{array} \text{---} \begin{array}{c} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \text{---} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} - \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \text{---} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right]$$

New rearrangement

Recent development: exotic systems $(q\bar{q})(q\bar{q})$

W. Heupel, G. Eichmann, C. S. Fischer, Phys. Lett. B718,545 (2012)



“famous double counting problem“)

The diagram illustrates the double counting problem in a perturbative expansion. It shows the following sequence of diagrams and equations:

- Top row: A diagram with two horizontal blue lines and two dashed blue lines meeting at a central vertex, connected by a vertical dashed blue line. This is equal to a diagram with a vertical red line passing through the vertex, which is equal to a diagram with a vertical red line passing through the vertex, but with the dashed lines crossing the red line.
- Bottom row: A sum of two diagrams (one with dashed lines on the left, one with dashed lines on the right) multiplied by another sum of two diagrams (one with dashed lines on the left, one with dashed lines on the right). This is equal to 2 times the original diagram plus an ellipsis.

$$\left(\begin{array}{c} \diagup \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \diagdown \\ \text{---} \\ \text{---} \end{array} \right) \times \left(\begin{array}{c} \diagdown \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \diagup \\ \text{---} \\ \text{---} \end{array} \right) = 2 \begin{array}{c} \diagup \\ \text{---} \\ \text{---} \end{array} + \dots$$

Gauge invariant currents; working out complicated combinatorics at the same time:

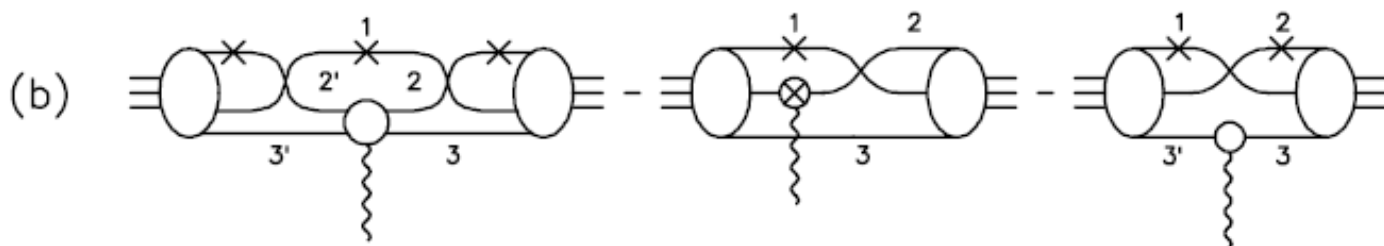
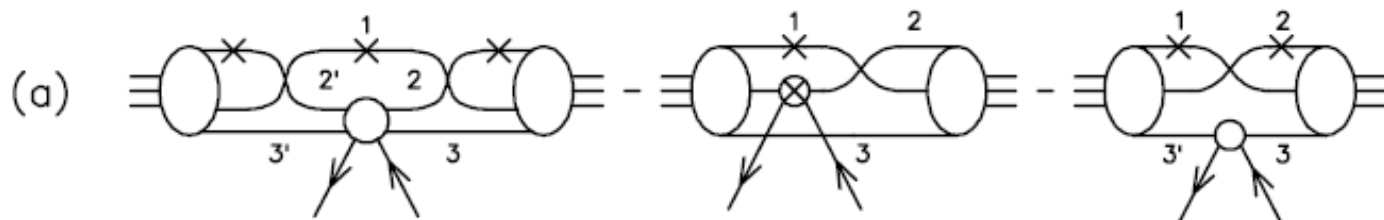
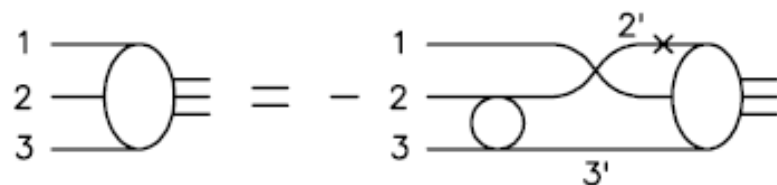
$$(\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0 \mathbf{V} \mathbf{G})_{\mu}$$

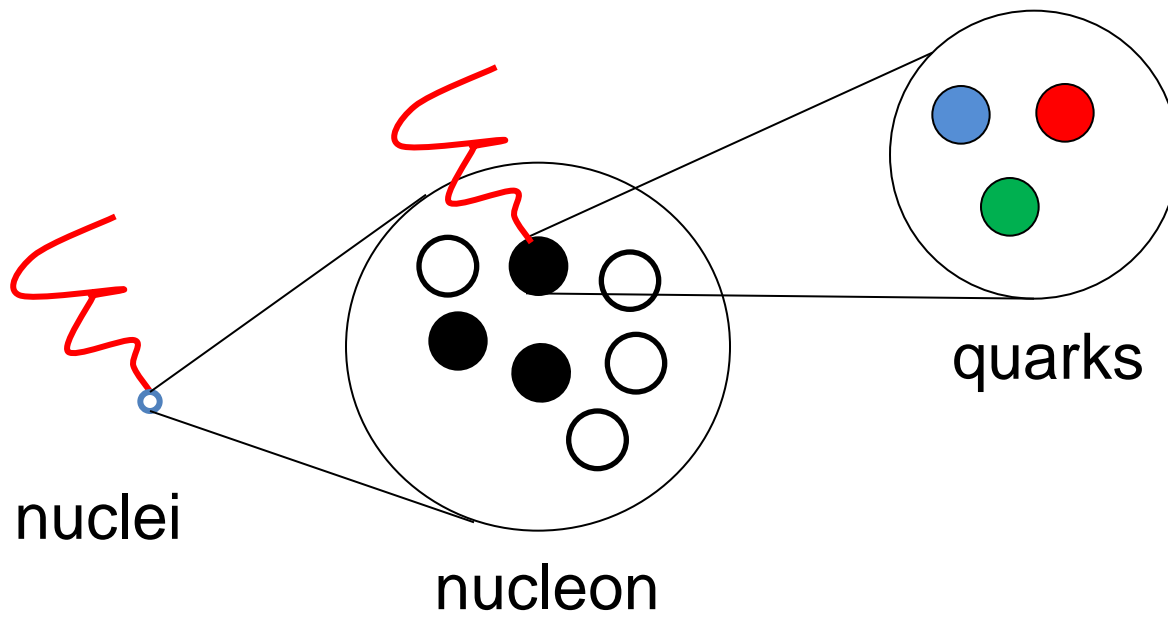
$$\mathbf{G}_{\mu} = \mathbf{G}_0 + \mathbf{G}_0_{\mu} \mathbf{V} \mathbf{G} + \mathbf{G}_0 \mathbf{V}_{\mu} \mathbf{G} + \mathbf{G}_0 \mathbf{V} \mathbf{G}_{\mu}$$

$$J_{\mu}(q) = \text{diagram} = \text{diagram} - \text{diagram}$$

The diagrammatic equation for the current $J_{\mu}(q)$ is shown. On the left, a blue circle with a red wavy line labeled q above it has two blue arrows pointing left from its left side. This is equal to the difference of two diagrams. The first diagram on the right shows two blue circles connected by two parallel blue lines, with a red wavy line labeled q above the top line. The second diagram on the right shows two blue circles connected by two parallel blue lines, with a red wavy line labeled q above the top line and a small vertical blue bar on the bottom line between the two circles.

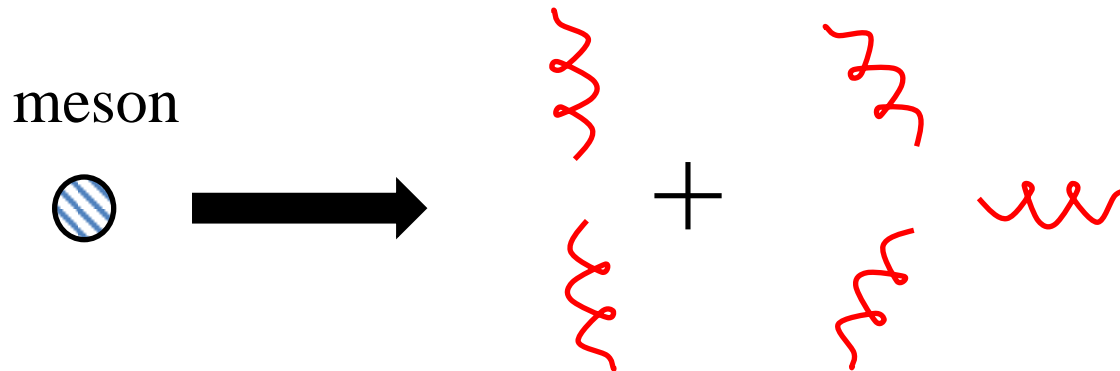
$$\Phi_1 = -t_1 d_2 d_3 P_{12} \Phi_1 \quad \rightarrow \quad \Phi_1 = -t_1 \delta_2 d_3 P_{12} \Phi_1$$





“Gauging equations” n-photon decay dynamics
from gauge invariance

Gauge Dyson-Schwinger equation few times
photons

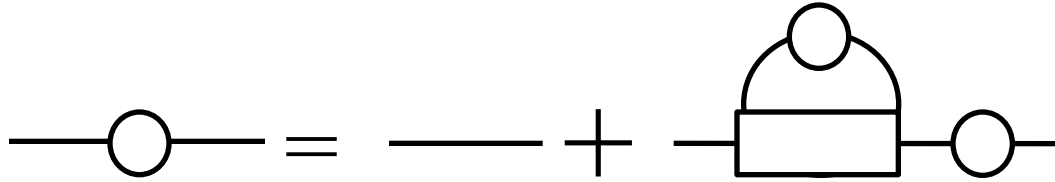


orthopositronium 3-photon decay - laboratory for
precision QED tests

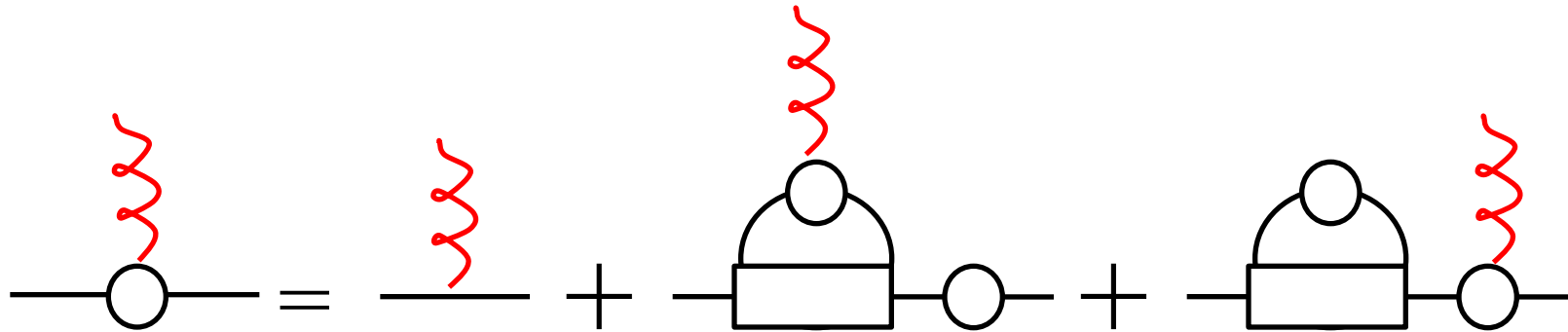
3-photon decays of vector quarkonia provide valuable
information about underlying QCD



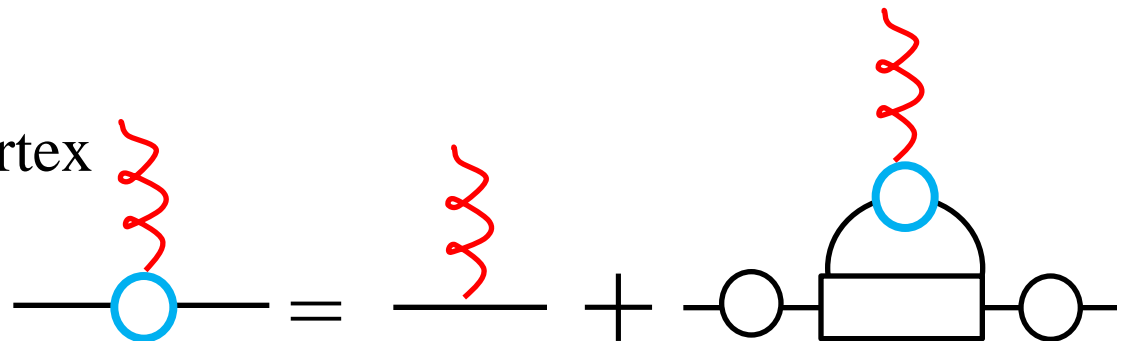
DS equation



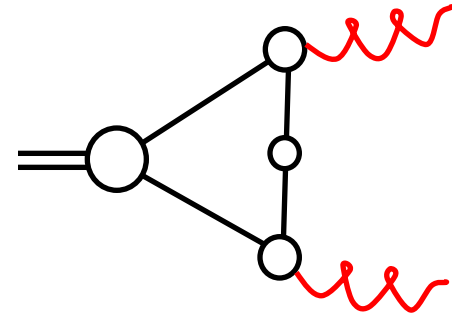
Single gauging of DS equation



Equation for γqq vertex

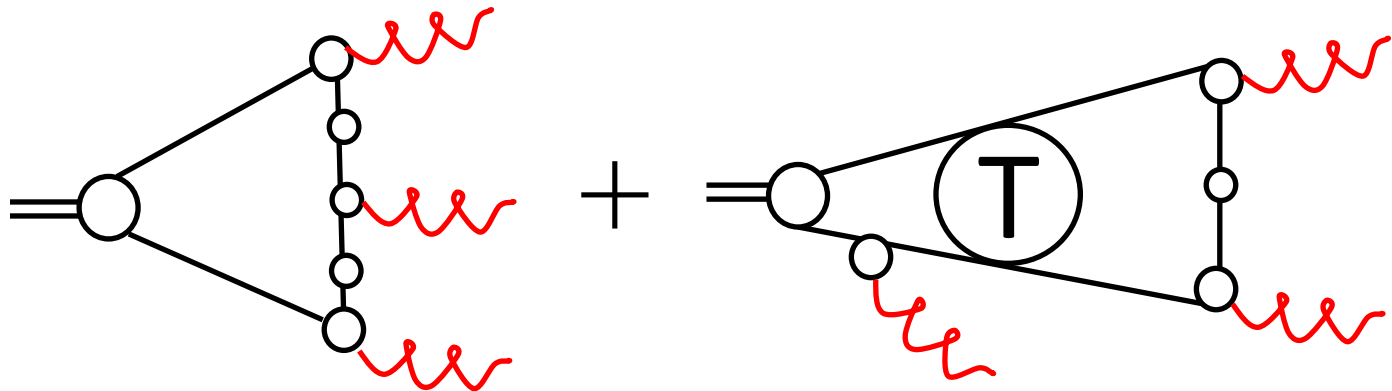


Double gauging of DS equation



2-photon decay amplitude

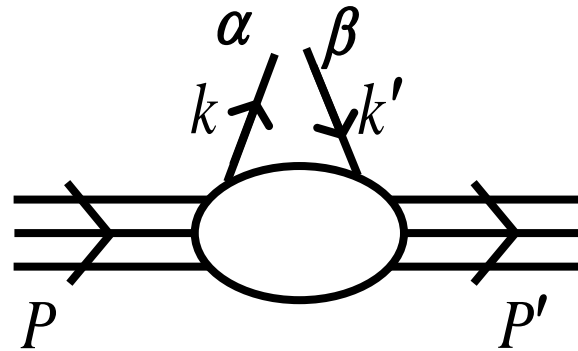
Triple gauging of DS equation



3-photon decay amplitude

Generalized Parton Distribution (GPD)

$$\rho_{\alpha\beta}(P', P, k) = \int d^4 y e^{iky} \langle P' | T \bar{q}_\beta(\mathbf{0}) q_\alpha(y) | P \rangle$$



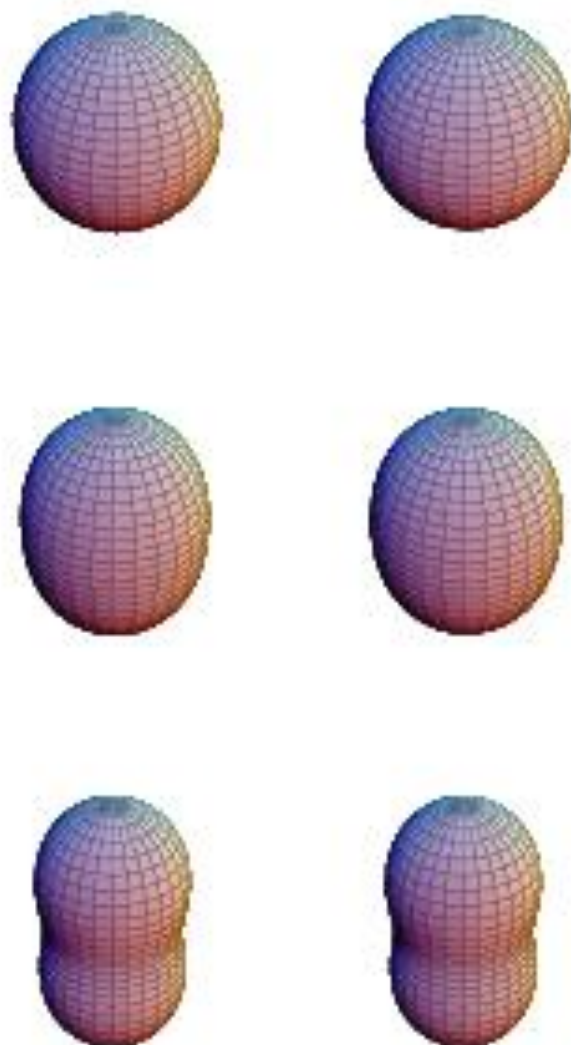


FIG. 1: Matter distributions. The spin direction \mathbf{s} is taken as vertical, $\hat{\mathbf{z}}$, with $\mathbf{n} = \hat{\mathbf{s}}$. Right column model of [7], left column model of [10]. First row $K = 250 \text{ MeV}/c$, second row $K = 1 \text{ GeV}/c$, third row $K \rightarrow \infty$.

On the Lagrangian level:

Gauge invariance

+finiteness of observables
(Renormalizability)

=Determines full dynamics